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A STRAIN BALANCE SYSTEM FOR ANALYSIS OF REDUNDANT STRUCTURES

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STRUCTURAL DIVISION

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A STRAIN BALANCE SYSTEM FOR ANALYSIS OF REDUNDANT STRUCTURES

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Quite often in the design of industrial structures there arise situations involving highly redundant frames. These frames may not appear to be very complex, but the determination of stresses by the ordinary methods at hand becomes very tedious if the structure is more than two or three times redundant. In fact, too often these structures are not even analysed because of their complexity, and simplifying assumptions such as "let the columns take the vertical loads and the bracing take the horizontal loads" are resorted to.

A frame typical of many refinery structures where heavy equipment loads are carried high above the ground is shown in Fig. 1. The loads considered are as shown, and the relative L/AE values are as tabulated in Table I. If it is assumed that the horizontal shear of 100k divides evenly between the bracing in each story, it will be found that the total vertical reactions for this structure will be 700k at C, 700k at A, and zero at B. But if an exact analysis is made of this frame, it will be found that the reactions are 840k at C, 420k at A, and 140k at B. This indicates that a more careful appraisal should be made of the interaction of the members.

To arrive at the theoretically correct reactions of 840k at C, etc., involves a considerable amount of labor. This frame is four times redundant, and some 300 or 400 separate computations, additions, entries, etc., are involved in the solution; a mistake in any one of these operations will result in an incorrect answer. And after the solution is obtained, it is often found that some of the member sizes should be changed, whereupon the whole analysis must be repeated if the correct stresses are desired. This is a discouraging prospect.

In order to arrive at stresses which are reasonably correct in a reasonable amount of time, we can make good use of two of the most simple and fundamental conditions applicable to structures: 1) The member forces at each joint must balance, and 2) The components of the structure must fit together after they have been stressed. In fact, if we have obtained a set of member forces by any means, and these two conditions are satisfied, then the forces are correct. So if we can assume a set of forces in the structure, plot the resultant deformation of the structure, and by an observation of this deformation plot arrive at a better approximation of the forces, we will have at hand a method whereby we can in very short order obtain a reasonably correct set of forces for the individual members.

Refer back to Fig. 1 again. Consider as redundants the diagonal members CD, FG, JK, and MN. If we assign to each of them an assumed

1. Chf. Civ. Engr., Ehrhart & Arthur, Inc., San Francisco, Calif.

force, then a preliminary set of forces follows immediately for all the rest of the members. For example, assume that each of these members carries 60% of the shear in its story, resulting in -85k force in each of these members. The forces in the remaining members are computed, and a table is made of the deformations corresponding to the forces in the members. See Table I - the deformations in the column headed δ_1 correspond to the forces in the column headed F_1. Note that only relative values of L/AE and δ have been used. The absolute values for L, A, and E could have been used, and as a result the actual deflection of the structure could be measured directly from the deflection diagram; but for finding the member forces, the relative values are sufficient.

After computing the deformations of the members, an ordinary Williot deflection diagram is drawn. Beginning with points A, B, and C of Fig. 1, however, it is apparent that the position of point D can be plotted in two different ways, either by plotting AD and CD deformations or by plotting AD and BD deformations. It is also apparent that when we have the proper forces in all the members, the two plots for D will coincide. Then proceeding further, it is found that point G can be plotted in two different ways from points D, E, F, and that the two plots for G must coincide when we have the correct solution. This also applies to points K and N. Note at this time that there were four conditions to be fulfilled in the plot, namely the coincidence of the two possible positions for each of the four points D, G, K, and N. In general, there will be the same number of conditions to be fulfilled in the plot as there are redundants in the structure.

Upon plotting the deformations δ_1 corresponding to the forces F_1 in the members AD, BD, and CD, we obtain the partial diagram of Fig. 2. From just this much of a diagram, we can immediately see that the forces assumed in the redundant members must be considerably revised in order to get the two points plotted for D to coincide. And by a more critical examination, we can also estimate the amount of revision required in the deformations, and hence the forces. In the figure, $D_{(C)}$ is the plotted point for the position of point D dependent on the deformations in AD and CD, and $D_{(B)}$ is the position of D with respect to δ_{AD} and δ_{BD}. From Fig. 2, it is evident that $D_{(C)}$ must be moved to the right and $D_{(B)}$ to the left, hence δ_{CD} is too small and δ_{BD} is too large. And it appears that δ_{CD} and hence the force in CD should be increased by perhaps 50% to make the two points plotted for D coincide. Also it is reasonable to increase the loads in the other three redundants by the same amount before arriving at a second set of forces, since all four of these diagonals will act in a similar manner.

Upon increasing the load in each of the redundants to -120k, obtaining the forces in the remaining members and the deformations of all the members, a second deflection diagram can be plotted as in Fig. 3. The forces obtained for the members are as shown in Table I in the column headed F_2 and the corresponding distortions are those listed in the column headed δ_2. This deflection plot is actually a superposition of two plots, one for the right hand structure ACMN and one for the left hand

structure ABLN. The points $G_{(C)}$, $K_{(C)}$, $N_{(C)}$, are the points G, K, C, in the right hand structure ACMN, and the points $G_{(B)}$, $K_{(B)}$, and $N_{(B)}$ apply to the left hand structure ABLN. So as to keep the diagram as simple as possible, all of the deflections of the members are not shown in the diagram. From Fig. 3, it is seen that there is a fair correspondence between the pairs of points plotted for D, G, K, and N, and that we are very much closer to the correct forces than we were on the first approximation. It can be seen in Fig. 3 that $N_{(B)}$ (the point N plotted up the left hand side of the structure) is to the right of $N_{(C)}$ (the point N plotted up the right hand side of the structure), while $K_{(B)}$ and $G_{(B)}$ are to the left of $K_{(C)}$ and $G_{(C)}$. This means that if the position of the left hand side of the structure (ABLN) is plotted independently of the right hand side of the structure (ACMN), it will be found that some of the points D, G, K, N in the left hand structure lie to the left of the points D, G, K, N in the right hand structure, and the rest of the points (point N) lies to the right. (Note that each side of the structure would be stable by itself, and hence will have a definite sidesway depending only on the deformations of the individual members). So the left hand side of the structure has deformed sideways about the same amount as the right hand side; thus no general change in the redundant forces is necessary, only slight local revisions.

To arrive at an even closer answer, it can be seen that δ_{MN} should increase and δ_{FG} should decrease somewhat, these points being the farthest from coincidence. But it can also be seen that $N_{(B)}$ has moved from its original point (when the structure was not loaded) very little more than $N_{(C)}$ has. Note again that the distance from point A, B, C, to point $N_{(B)}$ on the diagram of Fig. 3 is proportional to the actual movement of point N in the structure consisting of the fourteen members on the left hand side of Fig. 1 bounded by the points A, B, L, N, when those members are subjected to the loads in the column headed F_2 of Table I.

It will generally be found that when the plotted points have reached the correspondence shown in Fig. 3, the member forces are close enough for member sizing and revision. The actual forces in the redundants should be -135k for MN, -120k for JK, -109k for FG, and -115k for DC, so our assumption of -120k for each is only about 12% off in the worst case; and the loads in the vertical members near the bottom have a very close correspondence with the exact values, since they depend more on the sum of the bracing forces than on the individual bracing forces.

This method of approximating the redundant forces is of course immediately applicable to much higher structures of the type shown in Fig. 1. If we have a similar structure of say 10 stories, the procedure will be similar, although of course more lengthy. The objective will be to pick the diagonal loads so that the points plotted in the two different possible ways for the joints on the middle column (such as D, G, K, N in Fig. 3) will have a close correspondence with each other. When the

points on the middle column in the left hand side of the structure fall sometimes to the left and sometimes to the right of the corresponding points in the right hand side of the structure, a reasonably correct solution has been obtained, and the designer can rest assured that the maximum column loads are very close to those that would be found by exact methods.

When using this method of analysis to determine the member forces in structures where the reaction points are separated by more than two panels, the procedure of obtaining the deflection diagram is carried out similar to that used in the general case of unsymmetrical structures. A Williot diagram is first plotted by assuming one member fixed in direction; but instead of applying the Mohr correction as in the usual problem when the deflections as such are desired, the Williot diagram alone is sufficient for our purposes. Consider the structure shown in Fig. 4, which could be a truss used to lengthen out the column spacing. In this case, only two panels separate the reaction points, and the deflections can be plotted directly because it is known that the panel points E, A, and L will all fall on the same horizontal line. But if the truss is supported only at E and L, with no reaction point at A, then the Williot diagram may be plotted by the usual procedure that would be used if this structure were not redundant and the deflections were required to be found. If only the member forces are desired, however, it is not necessary to carry the Mohr rotation correction through; so the actual labor required to solve a problem of this nature is very little more than that required for a structure in which the deflections can be plotted directly.

This procedure of plotting a deflection diagram will also be found useful in checking structures that have been solved by other methods, such as virtual work or least work. The fact that these structures are statically indeterminate does not preclude their being checked with a deflection diagram. There are many chances for mistakes to creep into the analyses performed by these methods, and a check made by an entirely independent method is very reassuring.

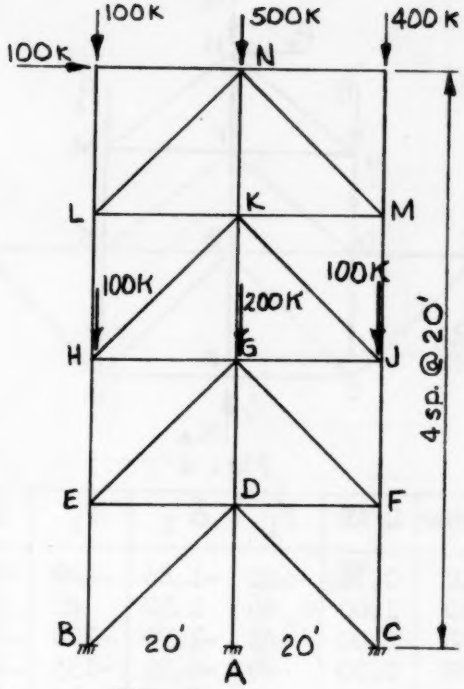

100K
500K
400K
100K
N
L
K
M
100K
200K
100K
H
G
J
4 sp. @ 20'
E
D
F
B
20'
20'
C
A

Fig. 1

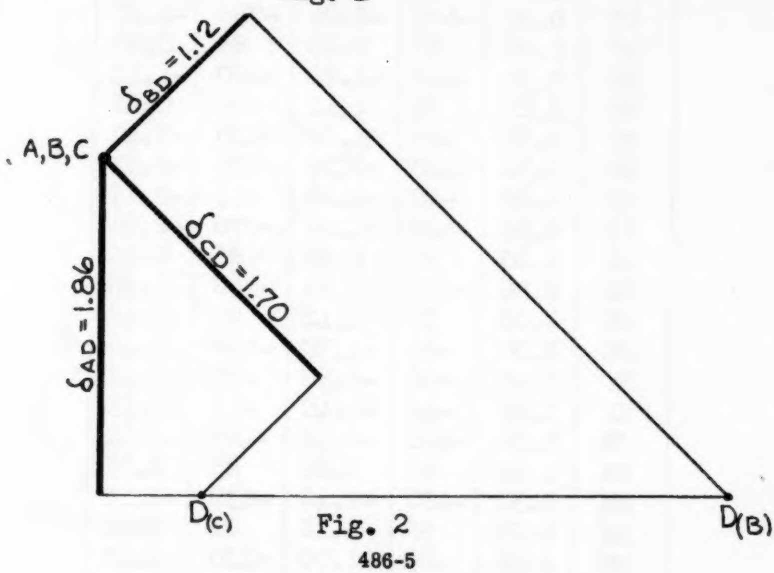

$\delta_{BD} = 1.12$
A,B,C
$\delta_{CD} = 1.70$
$\delta_{AD} = 1.86$
$D_{(C)}$
$D_{(B)}$

Fig. 2

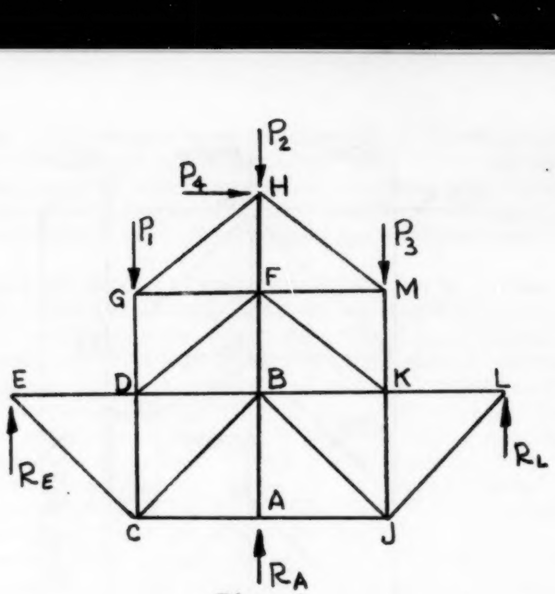
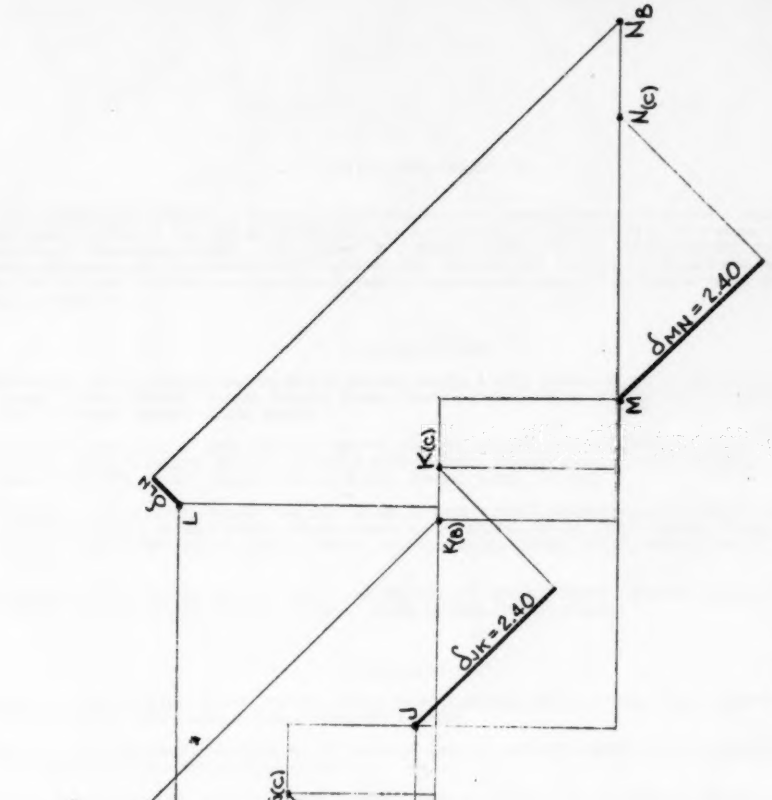


Fig. 4

Member	L/AE	F_1	δ_1	F_2	δ_2
AD	0.30	-620	-1.86	-420	-1.26
BD	2.00	56	1.12	21	0.42
CD	2.00	-85	-1.70	-120	-2.40
BE	0.30	-80	-0.24	-155	-0.45
DE	1.00	-40	-0.40	- 15	-0.15
CF	0.30	-680	-2.04	-755	-2.27
DF	1.00	60	0.60	85	0.85
DG	0.30	-640	-1.92	-490	-1.47
EG	2.00	56	1.12	21	0.42
FG	2.00	-85	-1.70	-120	-2.40
EH	0.30	-120	-0.36	-170	-0.51
GH	1.00	-40	-0.40	-15	-0.15
FJ	0.30	-620	-1.86	-670	-2.01
GJ	1.00	60	0.60	85	0.85
GK	0.50	-460	-2.30	-360	-1.80
HK	2.00	56	1.12	21	0.42
JK	2.00	-85	-1.70	-120	-2.40
HL	0.50	-60	-0.30	-85	-0.42
KL	1.00	-40	-0.40	-15	-0.15
JM	0.50	-460	-2.30	-485	-2.42
KM	1.00	60	0.60	85	0.85
KN	0.50	-480	-2.40	-430	-2.15
LN	2.00	56	1.12	21	0.42
MN	2.00	-85	-1.70	-120	-2.40

TABLE I



N_B
N(C)
δMN = 2.40
M
K(C)
K(B)
δJK = 2.40
J
L
δLN
G(C)
G(B)
δHK = .42
H
δFG = 2.40
F
E
δEG
D(C)
D(B)
δCD = 2.40
A,B,C
δBD
δAD = 1.26

Fig. 3

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a. Presented at the New York (N.Y.) Convention of the Society in October, 1953.
b. Beginning with "Proceedings-Separate No. 290," published in October, 1953, an automatic distribution of papers was inaugurated, as outlined in "Civil Engineering," June, 1953, page 66.
c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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